MODEL OF A FRACTURE AS AN EMITTER OF ELASTIC VIBRATIONS

L. A. Maslov

UDC 534.121.1:539.382.4

During the past two decades a new method has begun to be intensively developed for the investigation of fracture processes which is based on recording the mechanical vibrations generated by the defects of a medium [1]. The new method's problems include: extraction of a useful signal from the extraneous noises, identification of the type of defect, determination of its characteristic dimensions, and an estimate of the danger of the situation which has developed. The solution of the problems indicated has great meaning in such practical applications as nondestructive quality control and the engineering diagnostics of materials and manufactured goods. Therefore, the investigation of the spectrum of the signals produced by the formation of macroscopic fractures, such as the terminal and, consequently, most dangerous phase of fracture, is of great interest. The kinematical characteristics of a fracture as an emitter of elastic vibrations are formulated in this paper. The spatial and time spectra of the dynamical motions caused by the appearance of a growing fracture in a thin plate are discussed. Relationships are derived between the spectral characteristics of propagating disturbances and the parameters of the fracture.

§1. Let us assume that a two-dimensional stress state is realized, the material is isotropic, it is elastic right up to fracture, and the break is normal. The wave equations

$$\frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2}=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2};\quad \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2}=\frac{\partial\psi^2}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2} \tag{1.1}$$

with the initial and boundary conditions

$$u=v=du/dt=dv/dt=0,\ t=0; \tag{1.2}$$

$$u=0 \text{ when } |y|\geqslant l,\ x=0; \tag{1.3}$$

$$\sigma_x=q \text{ when } |y|\leqslant l,\ x=0, \tag{1.4}$$

occur in the medium, where u and v are the displacements along x and y, $2l$ is the fracture length, t is the time, φ and ψ are the longitudinal and transverse potentials, and c_1 and c_2 are their propagation rates (the potentials φ and ψ, the stresses, and the displacements are related by well-known functions [2]). The quantity q in the conditions (1.3) is the load (constant) acting on the fracture's edges in the direction of the x axis. In this case the principle of superposition of the stress–strain state is applied, which is justified by the linear statement of the problem. Thus, the problem of the propagation of "unloading" waves is replaced by the problem of the propagation of "loading" waves (the mirror-symmetric problem). However, the physical interpretation of the results obtained in the cited approach [the problem (1.1)-(1.3)] is rather complex. Another method of describing a developing defect is possible – the specification of its kinematical characteristics. One can write the boundary conditions (1.3) in such a case in the form

$$u=0 \text{ when } |y|\geqslant l,\ x=0; \tag{1.4}$$

$$u=u_0 \text{ when } |y|\leqslant l,\ x=0;$$

$$\tau_{xy}=0 \text{ when } |y|\leqslant\infty,\ x=0, \tag{1.5}$$

where u_0 is the kinematical source function. The initial conditions are written in the same form as in (1.2). Applying Laplace–Fourier (LF) transformations to Eqs. (1.1), we arrive at the usual second-order differential equations

Khabarovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 160–166, March-April, 1976. Original article submitted May 16, 1975.

$$\frac{d^2\bar{\varphi}}{dx^2} = \left(\omega^2 + p^2 c_1^{-2}\right)\bar{\varphi}; \tag{1.6}$$
$$\frac{d^2\bar{\psi}}{dx^2} = \left(\omega^2 + p^2 c_2^{-2}\right)\bar{\psi},$$

where $p = \alpha + i\gamma$ is the parameter of the Laplace transformation with respect to the time t and ω is the parameter of the Fourier transformation with respect to the coordinate y. The general solution of Eqs. (1.6) is of the form

$$\bar{\varphi} = Ae^{-x\sqrt{\omega^2+c_1^{-2}p^2}} + \tilde{A}e^{x\sqrt{\omega^2+c_1^{-2}p^2}};$$
$$\bar{\psi} = Be^{-x\sqrt{\omega^2+c_2^{-2}p^2}} + \tilde{B}e^{x\sqrt{\omega^2+c_2^{-2}p^2}}.$$

Assuming that $\bar{\varphi}$ and $\bar{\psi} \to 0$ as $x \to 0$, we obtain $\tilde{A} = \tilde{B} = 0$.

Let us apply the LF transformation to the boundary conditions (1.4) and (1.5) of the problem. Condition (1.4) is written in the form

$$-A\sqrt{\omega^2 + c_1^{-2}p^2} + i\omega B = \bar{u}_0, \tag{1.7}$$

and the condition (1.5) is written as

$$i\omega A\sqrt{\omega^2 + c_1^{-2}p^2} + \left(\omega^2 + \frac{1}{2}c_2^{-2}\right)B = 0. \tag{1.8}$$

Solving (1.7) and (1.8) simultaneously, we find

$$A = -\bar{u}_0 \frac{2\omega^2 + c_2^{-2}p^2}{c_2^{-2}p^2\left(\omega^2 + c_1^{-2}p^2\right)^{1/2}},$$
$$B = \bar{u}_0 \frac{2i\omega}{c_2^{-2}p^2}.$$

Hence the solution being sought is written in the form

$$\bar{\varphi} = -\bar{u}_0 \frac{2\omega^2 + c_2^{-2}p^2}{c_2^{-2}p^2\left(\omega^2 + c_1^{-2}p^2\right)^{1/2}} e^{-x\sqrt{\omega^2+c_1^{-2}p^2}};$$
$$\bar{\psi} = \bar{u}_0 \frac{2i\omega}{c_2^{-2}p^2} e^{-x\sqrt{\omega^2+c_2^{-2}p^2}}.$$

Converting to the displacements u, v, and w, we obtain their spectra

$$\bar{u} = \bar{u}_0\left(\frac{2\omega^2 + c_2^{-2}p^2}{c_2^{-2}p^2} e^{-x\sqrt{\omega^2+c_1^{-2}p^2}} - \frac{2\omega^2}{c_2^{-2}p^2} e^{-x\sqrt{\omega^2+c_2^{-2}p^2}}\right); \tag{1.9}$$
$$\bar{v} = \bar{u}_0\left(-i\omega\frac{2\omega^2 + c_2^{-2}p^2}{c_2^{-2}p^2\left(\omega^2 + c_1^{-2}p^2\right)^{1/2}} e^{-x\sqrt{\omega^2+c_1^{-2}p^2}} + \frac{2i\omega\left(\omega^2 + c_2^{-2}p^2\right)^{1/2}}{c_2^{-2}p^2} e^{-x\sqrt{\omega^2+c_2^{-2}p^2}}\right);$$
$$\bar{w} = \bar{u}_0\left(\frac{\mu d}{2(1-\mu)}c_1^{-2}p^2 \frac{2\omega^2 + c_2^{-2}p^2}{c_2^{-2}p^2\left(\omega^2 + c_1^{-2}p^2\right)^{1/2}} e^{-x\sqrt{\omega^2+c_1^{-2}p^2}}\right).$$

The displacements w are expressed in terms of u and v with the hypothesis of a two-dimensional stress state taken into account by the relationship

$$w = -[\mu d/2(1-\mu)](\partial u/\partial x + \partial v/\partial y),$$

where d is the plate's thickness. It is easy to see from the relationships (1.9) that it is possible by measuring the spectra of the displacements u, v, or w to determine the function $\bar{u}_0$, and applying the inverse LF transformation, establish the physical process of the fracture's formation. In addition, the function $\bar{u}_0$ is a modulating function with respect to the expressions contained within parentheses in (1.9). Consequently, its zeroes (or extrema) will also be zeroes (extrema) of the functions $\bar{u}$, $\bar{v}$, and $\bar{w}$.

Thus, the problem reduces to a search for the function u_0, i.e., to the construction of a model of a fracture as an emitter of elastic vibrations. However, simulation of a fracture on the basis of experimental data is not possible for a number of reasons (for example, its high velocity and the small amount of the fracture's opening). Therefore, the investigation of the behavior of a fracture as the source of elastic vibrations was carried out with the aid of a computer. Problem (1.1)-(1.3) was reduced to a finite-difference scheme, programmed in the Algol-60 language, and solved on a Minsk-22M electronic computer. Central-difference equations of the first approximation were used to represent derivatives. Integration was performed according to an explicit three-layer scheme. Special attention was paid to the choice of parameters which provide for the method's stability and the computational process' convergence [3].

§2. The following problem in the arrangement (1.1)-(1.3) has been solved by the numerical method: the formation of an internal fracture of constant length L in a rectangular plate of finite dimensions whose edges are rigidly restrained. The following parameters were varied: constants of the material, the plate's dimensions, the load on the fracture's edges, and its length. The solution made it possible to study the development of the wave process caused by the formation of the defect. The temporal and spatial forms of the fracture's opening were given consideration in the analysis of the numerical solution as the direct source of the disturbances. Taking account of the plate's finite dimensions, numerical data were analyzed in the time interval from the instant of disruption of the strong forces among the material's particles (in the mirror problem – from the instant of application of the load q) to the time corresponding to the arrival of the wave reflected from the plate's boundary. As a result it proved possible to represent the quantity u_0 – the kinematical source function – to a high degree of accuracy by a function of the form

$$u_0 = \begin{cases} st \text{ when } 0 \leqslant t \leqslant T(y) \\ sT(y) \text{ when } t \geqslant T(y), \end{cases} \tag{2.1}$$

where $T(y) = (2/c_1)(l^2 - y^2)^{1/2}$, $s = q/\rho c_1$, and ρ is the material's density.

The time dependence of the shape of the motion of point O of the fracture's surface is shown in Fig. 1 as curve 1: curve 2 depicts the function u_0 at $y = 0$. The calculation was carried out for the following values of the initial parameters: $q = 500$ kg/cm², $L = 0.4$ cm, $c_1 = 4.9 \cdot 10^5$ cm/sec, and $\rho = 0.78 \cdot 10^{-5}$ kg sec²/cm⁴. The geometrical shape of the fracture's opening (curve 1) is shown in Fig. 2 at the instants $\bar{t} = 2$ and $\bar{t} = tc_1/a$, where $a = 1.4$ cm is the distance from the fracture's axis to the plate's edge; curve 2 depicts the size of the fracture's static opening determined by Westergaard's equation [4]. It is evident from the data cited that the problem of the formation of a fracture can be considered as transitional from an initial no-defect state to a state of static equilibrium defined by Westergaard's equation. The time of the fracture's opening T (the interval of the source's activity) is determined by its length and does not depend on the size of the stresses q, which determine the rate s of the fracture's opening (or the steepness of the leading edge of the source's mechanical pulse). The results of the analysis conducted are expressed by the relations (2.1). Applying the LF transformation to (2.1), we obtain

$$\overline{u_0} = \frac{q}{\rho c_1^2} \int_{-l}^{+l} \frac{1}{p^2} \left(1 - e^{-pT(y)}\right) e^{i\omega y}\, dy. \tag{2.2}$$

We will adopt $y = 0$ in the expression for $T(y)$, i.e., we will set $T(y) = L/c_1$ (the latter is equivalent to the replacement of an elliptical shape for the fracture's opening by a rectangular one) to further simplify the analysis. Then it follows from (2.2) that

$$\overline{u_0} = \frac{q}{i\rho c_1^2 p\omega} R(p) Q(\omega), \tag{2.3}$$

where $R(p) = 1 - e^{-pL/c_1}$ is a factor produced by the temporal form of the fracture's opening and $Q(\omega) = e^{i\omega l} - e^{-i\omega l}$ is a factor produced by the fracture's geometry and its linear extent. Taking account of what has been stated above, one can see that the zeroes of the modulus of the investigated signal's spectral density, which are determined by the factor $R(p)$, are arranged over the frequency interval

$$\Delta f = c_1/L. \tag{2.4}$$

Here $f = \gamma/2\pi$. [Since the integral (2.2) is convergent when $\alpha > 0$, $\alpha \to 0$ is assumed in the derivation of (2.4) from the relation (2.3).] Hence, it follows that one can determine the defect's length by measuring the spectrum of the acoustical emission signals caused by fracture formation.

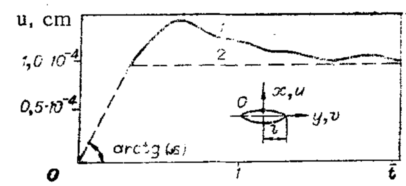

Fig. 1

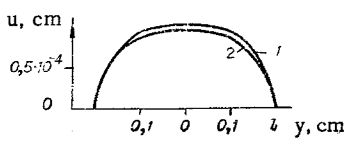

Fig. 2

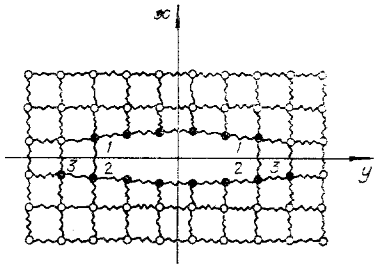

Fig. 3

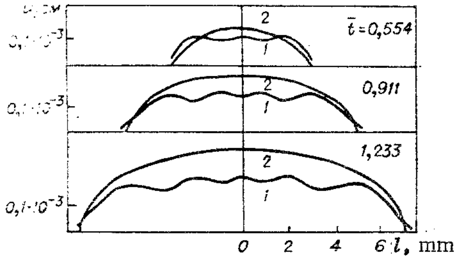

Fig. 4

One can determine the fracture's length by measuring the spatial spectrum of the mechanical vibrations caused by the factor $Q(\omega)$ and obtain a relation analogous to (2.4). However, taking account of the fact that ω in the adopted system of notation is the expansion parameter with respect to the coordinate y, we arrive at the necessity of applying several transducers included in parallel to establish the spatial spectrum instead of the single one used for determining the time spectrum. Analysis of the expression $Q(\omega)$ allows one to conclude that the directional nature of the emission from the fracture is analogous to the directionality of electromagnetic radiation from a short-wavelength vibrator, which is very important in the selection of the most rational mounting position for the recording transducers.

Considering the frequency interval between $f = 0$ and the first zero of the spectrum, one can say that its width is equal to Δf. Taking the approximate relation $\Delta f T \approx 1$ into account, we obtain from (2.4) a relation between a certain temporal characteristic of the propagating signal and the fracture's length

$$L = c_1 T. \qquad (2.5)$$

The correctness of the last relation is confirmed upon analysis of the results of the numerical solution.

It is interesting to note the following. The problem of the dynamical stability of fractures which exist in a brittle material has been discussed in [5] in the case of the action of pulse loads on this material, and it has been shown that the fracture whose length L, pulse duration, and speed of sound in the material c are related by the equation

$$L = cT$$

turns out to be unstable upon the propagation of a square stress pulse of duration T. The notion of the fracture as a gorge of some effective thickness was used by the authors in solving the indicated problem. General assumptions were advanced in the paper [6] as to the temporal form of the displacements caused by the fracture. On the basis of an assumption as to the temporal form of the radiation from the fracture, the authors of [6] came to corresponding conclusions as to the nature of the spectrum of the emitted signals. The aspect of a fracture as a reflector was also pointed out in the paper [7]: a fracture of length L completely reflects a vibration having frequency $f \geq c_1/L$.

The numerical solution of the problem of the fracturing of a continuity of constant length has permitted obtaining a transient moving fracture. The investigation of the dynamical stresses and distortion energy U at the fracture's apex has shown that these quantities are proportional to the known constants of fracture mechanics – the stress-intensity coefficient k_1 and the specific surface energy γ.

Taking account of the concentration of stresses σ_x and the energy U at the fracture's apex, and also their proportionality to k_1 and γ, the solution of the transient problem has been carried out in the following way. Upon the fulfillment of any of the conditions: $\sigma_x \geq \sigma_x^*$ and $U \geq \sigma^*$ (σ_x^* and U^* are specified critical values), an increase has occurred in the fracture's length in each direction by the amount of a grid step. The average velocity was calculated from the instant of the preceding jump and was referred to the middle of the time interval. Investigations have shown that the discrete model of fracture shown in Fig. 3 evidently gives the most correct idea of the fracture's growth, which is in agreement with known analytic estimates. In this model points of the finite-difference grid represent material particles among which interactive forces occur for binding. The stresses at the fracture's apex were calculated from well-known equations [2] by means of the approximation of derivatives with respect to the displacements at the points 1, 2, and 3 (see Fig. 3). When the values of σ_x^* and U^* are near and close to q and U_{st} – the distortion energy corresponding to a static opening of the fracture – the limiting velocities amount to $V = 0.71\ c_2$ and $V = 0.68\ c_2$, respectively.

The opening of the moving fracture was investigated in the process of solving the transient problem. The shape of a fracture's surface whose development was determined by the energy criterion ($U \geq U^*$) is

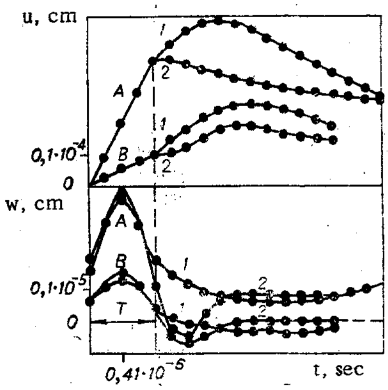

Fig. 5

shown in Fig. 4. Curves 1 are the displacements u of the fracture's surface, and curves 2 are the shape of a static fracture of corresponding length. It is obvious here that the fracture's surface has a shape close to elliptical, and the opening at each instant of time corresponds to the instantaneous value of its length. The ratio of the semiminor axes of the static and dynamical fractures amounts to 1.2-1.4 for velocities close to the limiting value.

Hence, one can write the kinematical function of a moving fracture approximately in the form

$$u_0 = \begin{cases} \frac{2\rho q}{c_1^2}(l^2(t) - y^2)^{1/2}, & 0 \leq t \leq T \\ \frac{2\rho q}{c_1^2}(l^2(T) - y^2)^{1/2}, & t \geq T, \end{cases} \tag{2.6}$$

where T is the time of the jump. Applying the LF transformation to (2.6), we get

$$\overline{u_0} = \frac{2\rho q \pi V}{c_1^2 \omega}\left(\int_0^T e^{-pT} t I_1\left(\frac{\omega V t}{2}\right) dt + \frac{T}{p} I_1\left(\frac{\omega V T}{2}\right) e^{-pT}\right),$$

where V is the jump's velocity (constant) and I_1 is the cylindrical function of the first kind. Hence, as $\omega \to 0$ (for spatial harmonics close to zeroth order) and $\alpha \to 0$, we find a dependence analogous to (2.4) between the time of the fracture's jump and the distribution of the time spectrum's extrema,

$$\Delta f = 1/T. \tag{2.7}$$

The model formulated for a fracture as an emitter (2.1) has been realized in a numerical solution. Comparison of the exact solution and the solution based on the adopted model is given in Fig. 5. Here curves 1 are the displacements of the points A (0.5 a; 0) and B (0.5 a; 0.3 a) of the plate obtained by solving the problem with specified loads on the fracture surface, and curves 2 are the displacements at those same points for a specified law of the fracture's opening.

A procedure has been cited in the paper [8] for measuring the spectrum of the acoustical emission signals for individual jumps of the fracture. The results of an experiment showed that the location of the first minimum relative to the frequency depends on the duration of the jump in the rectangularly developing fracture, which is confirmed by the results of this paper [Eq (2.7)].

The author thanks L. I. Slepyan for valuable discussion and attention to this research.

LITERATURE CITED

1. Yu. I. Bolotin, V. A. Greshnikov, A. A. Gusakov, Yu. B. Drobot, and V. P. Chentsov, "Investigation of the emission of stress waves for the nondestructive control of materials and manufactured goods," Defektoskopiya, No. 6 (1971).
2. V. V. Novozhilov, The Theory of Elasticity [in Russian], Sudpromgiz, Leningrad (1958).
3. L. I. Dyatlovitskii, A stable explicit difference scheme for solving Lamé equations of motion (planar case)," Dokl. Akad. Nauk URSR, Ser. A, No. 12 (1968).
4. H. M. Westergaard, "Bearing pressure and cracks," J. Appl. Mech., 6, No. 2 (1939).
5. B. Steverding and S. H. Lehnigk, "Response of cracks to impact," J. Appl. Phys., 41, No. 5 (1970).
6. R. W. Stephens and A. A. Pollock, "Waveforms and frequency spectra of acoustic emissions," J. Acoust. Soc. Amer. (JASA), 50, No. 3 (1971).

7. G. P. Cherepanov, The Mechanics of Brittle Fracture [in Russian], Nauka, Moscow (1974).
8. Yu. I. Lykov, L. A. Maslov, and V. I. Panin, "Determination of the duration of a fracture's jump by measuring the spectrum of the acoustical emission signals," Defektoskopiya, No. 6 (1974).